Network inference

Gaussian Graphical models
- Generative model of the signal
- Interaction between regions estimated by partial correlation
- Amounts to covariance estimation

An estimation problem
- Many brain regions, short time series
- Inter-subject variability prevents data accumulation

\[ \ell_{21} \text{ penalization for inverse covariance} \]

\[
\left( \hat{K}^{(s)}_{\ell_{21}} \right)_{s=1..N} = \text{arg min} \sum_{s=1}^{S} \left( \text{tr}(K^{(s)} S_{\text{sample}}^{(s)}) - \log \det K^{(s)}) + \lambda \sum_{i \neq j} \|K^{(s)}_{ij}\|_2 \right)
\]

- Joint sparsity: pattern shared in population (similar to group-lasso)
- Convex optimization with cyclical coordinate descent on Choleski decompositions of the precision matrices [A. Rothman, 2008]

Subject-level edge values

Group-level edge selection

Experimental validation

Use a full-brain atlas to extract time-series
- Probabilistic atlas of anatomical structures (poster 335)
- 137 cortical and sub-cortical regions

Resulting sparse precision matrices

Cross validation results

Comparison with other covariance estimation method:
- LW = Ledoit-Wolf: non-sparse shrinkage
- \( \ell_1 = \) Normal sparse inverse covariance

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<th>Using subject data</th>
<th>Uniform group model</th>
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Reference:
G. Varoquaux et al., Brain covariance selection: better individual functional connectivity models using population prior, Adv. NIPS 2010
http://books.nips.cc/nips23.html